In conclusion, we will present some of the dimensional quantities corresponding to plate motion with $Re^* = 3 \cdot 10^3$ in an electrolyte with $\sigma = 10^{12}$ 1/sec. Let $2a = 10^2$ cm. Then from Eq. (2.6) at $\delta = 0.4$ it follows that $\omega = 9 \cdot 10^4$ 1/sec, and from Eq. (4.4) and $v_0/u_0 = 0.39$ we have $v_0 = 1.2 \cdot 10^{-1}$ cm/sec; from Eq. (4.3), (2.5) we determine the amplitude of the maximum magnetic field intensity $H_0 = 2\pi I_0/\sec \simeq 2.7$ G.

LITERATURE CITED

1. I. M. Kirko, Liquid Metal in a Magnetic Field [in Russian], Énergiya, Moscow–Leningrad (1964).
2. O. M. Phillips, "Prospects for magnetohydrodynamic ship propulsion," J. Ship Res., 5, No. 4 (1962).
3. L. G. Vasil'ev and A. I. Khozhainov, Magnetohydrodynamics in Ship Technology [in Russian], Sudostroenie, Leningrad (1967).
4. V. I. Merkulov, "Motion of a sphere in a conductive liquid in the presence of crossed electric and magnetic fields," Magn. Gidrodin., No. 1 (1973).
5. A. Sneyd, "Generation of fluid motion in a circular cylinder by an unsteady applied magnetic field," J. Fluid Mech., 49, 4 (1971).
6. K. B. Dzhakupov, "Some difference schemes for the Navier–Stokes equations", Ch. M. M. S. S., 2, No. 1 (1971).
7. T. V. Kuskova, "Difference method for calculation of the flow of a viscous incompressible liquid," in: Calculation Methods and Programming [in Russian], 7th ed., Moscow State Univ. (1967).
8. V. I. Merkulov, V. I. Panchuk, and V. F. Tkachenko, "Solution of motion equations of an electrically conductive liquid in a longitudinal traveling magnetic field," in: Proceedings of the All-Union Seminar on Numerical Methods in Mechanics of a Viscous Liquid [in Russian], Nauka, Moscow (1969).
9. Tsan Sue-Sen, "The Poincaret–Lighthill–Ho Method," in: Problems in Mechanics [Russian translation], IL, Moscow (1959).
10. Z. Janour, "Resistance of a plate in a parallel flow at low Reynolds number," NACA TM No. 1316 (1951).

UNSTEADY FLOW OF A NON-NEWTONIAN LIQUID WITH A POWER RHEOLOGICAL LAW PAST A FLAT PENETRABLE PLATE

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We analyze the problem of the unsteady flow of a non-Newtonian liquid with a power rheological law past a flat penetrable plate. In contrast with [1], where a similarly posed problem is treated for a pseudoplastic liquid, we solve the problem for a dilatant liquid.

For a non-Newtonian liquid with a power rheological law, the relation between the shear stress τ and the velocity gradient $\partial u/\partial z$ for plane motion has the form [2]

$$\tau = k\left|\frac{\partial u}{\partial z}\right|^{n-1}\frac{\partial u}{\partial z} \quad (n > 0),$$

where k and n are rheological constants of the medium; the case $n = 1$ corresponds to Newtonian liquid, $n < 1$ to a pseudoplastic liquid, and $n > 1$ to a dilatant liquid.

The problem of the flow of a non-Newtonian liquid with a power rheological law past an infinite flat plate in the presence of uniform suction of liquid depending on time according to a definite law was treated in [1]. This problem was solved for pseudoplastic ($n < 1$) and Newtonian ($n = 1$) liquids. We solve the problem in a similar formulation without restriction on the possible values of $n > 0$.

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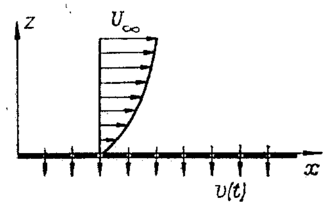

Fig. 1

We consider the unsteady flow of a power-law liquid past a flat surface z = 0 (Fig. 1) across which there is a transverse motion of liquid with the velocity

$$v(t)=\frac{\beta}{n+1}\left(\frac{U_\infty^{1-n}}{na}\right)^{-\frac{1}{n+1}}(t+t_0)^{-\frac{n}{n+1}}, \tag{1}$$

where β = const is the velocity of the incoming flow; t, time; t_0, initial time; and $a = k/\rho$.

In the boundary-layer theory approximation the equation of the nongradient motion of a power-law liquid can be written in the form

$$\frac{\partial u}{\partial t}+v(t)\frac{\partial u}{\partial z}=a\frac{\partial}{\partial z}\left(\frac{\partial u}{\partial z}\right)^n. \tag{2}$$

The problem of the existence and general character of the behavior of the solution of an equation of the type (2) is covered in detail in [3, 4].

An exact analytic solution of Eq. (2) for an arbitrary law of motion of the liquid v(t) has not been obtained, but an exact self-similar solution of Eq. (1) exists.

The boundary conditions in the case under consideration are

$$u(0)=0,\ u(\infty)=U_\infty. \tag{3}$$

The self-similar variable η and the velocity u are written in the form

$$\eta=\left(\frac{U_\infty^{1-n}}{na}\right)^{\frac{1}{n+1}}(t+t_0)^{-\frac{1}{n+1}}z,\quad u=U_\infty f(\eta). \tag{4}$$

By using (1) and (4) the solution of (2) and (3) can be reduced to the integration of the ordinary differential equation

$$\frac{n-1}{n+1}(\beta-\eta)=\frac{d}{d\eta}\left(\frac{df}{d\eta}\right)^{n-1} \tag{5}$$

with the boundary conditions

$$f(0)=0,\ f(\infty)=1. \tag{6}$$

Integrating (5) twice, we obtain

$$f(\eta)=\left[\frac{n-1}{2(n+1)}\right]^{\frac{1}{n-1}}\int_0^\eta(2\beta\eta-\eta^2+\text{const})^{\frac{1}{n-1}}d\eta. \tag{7}$$

The solution of Eq. (5) is essentially different for n < 1 and n > 1, and therefore we treat these cases separately.

For n < 1 it is convenient to rewrite Eq. (7) in the form

$$f(\eta)=\left[\frac{1-n}{2(1+n)}\right]^{\frac{1}{n-1}}\int_0^\eta[(\eta-\beta)^2+A^2]^{\frac{1}{n-1}}d\eta,$$

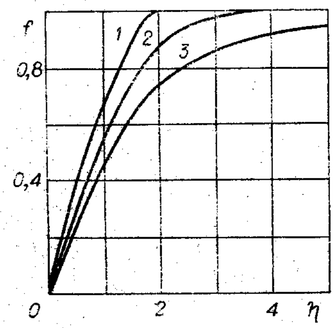

Fig. 2

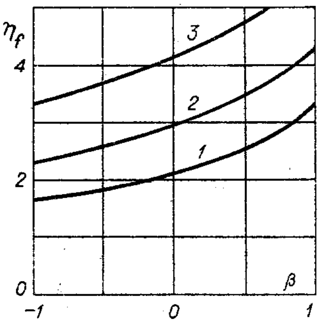

Fig. 3

where A is a constant to be determined.

By introducing the new variables $\xi = \eta - \beta$, and using boundary condition (6), we obtain

$$\int_{-\beta}^{\infty} (\xi^2 + A^2)^{\frac{1}{n-1}} d\xi = \left[\frac{2(1+n)}{1-n}\right]^{\frac{1}{n-1}}. \tag{8}$$

Evaluating the integral on the left-hand side of (8), we obtain a transcendental equation for the constant A

$$\frac{1}{2} A^{\frac{n+1}{n-1}} B\left[\frac{1}{2}, \frac{n+1}{2(1-n)}\right] + \frac{1}{2} A^{\frac{2}{n-1}} \beta B\left(1, \frac{1}{2}\right) F\left(-\frac{1}{n-1}, \frac{1}{2}, \frac{3}{2}, -\frac{\beta^2}{A^2}\right) = \left[\frac{2(1+n)}{1-n}\right]^{\frac{1}{n-1}},$$

where B (p, q) is the beta function and F (α, β, γ, z) is the hypergeometric function.

For $n > 1$ we have

$$f(\eta) = \left[\frac{(n-1)}{2(n+1)}\right]^{\frac{1}{n-1}} \int_0^{\eta} [C^2 - (\eta - \beta)^2]^{\frac{1}{n-1}} d\eta, \tag{9}$$

where C is a constant to be determined.

Study of (9) shows that a limiting value $\eta = \eta_f$ exists such that for all $\eta > \eta_f$ $f(\eta) = 1$, i.e., shear perturbations for $n > 1$ are localized within a finite distance from the surface of the plate.

By introducing the new variable

$$(y - \beta)^2 = C^2,$$

we obtain two supplementary conditions:

$$f(\eta) = 1, \ df/d\eta|_{\eta=y} = 0. \tag{10}$$

Taking account of (10), we have

$$\int_0^{y} [(y-\beta)^2 - (\eta - \beta)^2]^{\frac{1}{n-1}} d\eta = \left[\frac{2(n+1)}{n-1}\right]^{\frac{1}{n-1}} \tag{11}$$

Introducing the variables $\theta = \eta - \beta$ and $v = z - \beta$, and evaluating the integral in (11), we find an equation for C

$$y^{\frac{n+1}{n-1}}\left[\frac{1}{2}B\left(\frac{1}{2},\frac{n}{n+1}\right)+2^{\frac{n+1}{n-1}}\left\{B_{\frac{y+\beta}{y}}\left(\frac{n}{n-1},\frac{n}{n-1}\right)-B_{\frac{1}{2}}\left(\frac{n}{n-1},\frac{n}{n-1}\right)\right\}\right]=\left[\frac{2(n+1)}{n-1}\right]^{\frac{1}{n-1}},$$

where $B_p(p, q)$ is the incomplete beta function.

Results calculated with the equations derived are shown in Figs. 2 and 3. Figure 2 shows dimensionless velocity profiles for various values of n and the injection parameter (suction) β [1) n = 2; 2) 1.25; 3) 0.5]. Figure 3 shows the position of the front of the shear perturbations as a function of the injection parameter (suction) β for various values of the rheological constant n [1) n = 2; 2) 1.25; 3) 1.15].

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LITERATURE CITED

1. Z. P. Shul'man and B. M. Berkovskii, The Boundary Layer of Non-Newtonian Liquids [in Russian], Nauka i Tekhnika, Minsk (1966).
2. W. Wilkinson, Non-Newtonian Fluids, Pergamon, New York (1960).
3. O. A. Oleinik, "A system of equations of boundary-layer theory," Zh. Vychisl. Mat. Mat. Fiz., No. 3 (1963).
4. A. I. Suslov, "A system of equations of a boundary layer with a surface of discontinuity," Usp. Mat. Nauk, 29, No. 3 (1974).

VISCOUS EXPLOSION DURING THE NONISOTHERMAL MOTION OF AN INCOMPRESSIBLE LIQUID

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The hydrodynamic thermal explosion of an incompressible liquid moving under pressure in pipes was predicted theoretically in [1-3].

We present the hydraulic theory of a viscous explosion, which is caused by the nonlinear temperature dependence of viscosity.

Let us consider the laminar motion of an incompressible liquid in a circular pipe of radius R and length L. The pressure is p_1 at the pipe inlet and p_2 at the outlet. The temperature of the liquid at the inlet cross section is T_0, and a steady heat flux $\lambda\partial T/\partial r = q_w < 0$ is specified at the pipe walls (heat is removed from the liquid). The physical quantities λ, ρ, and C_p are assumed constant in the temperature range considered.

It is assumed that the Peclet number $Pe = uR\rho C_p/\lambda \gg 1$, so that axial heat conduction can be neglected in the heat-balance equation. We linearize the convective terms of this equation in the following way [4]:

$$\mathbf{v}\,\mathrm{grad}\,T \approx \frac{Q}{\pi R^2}\frac{\partial T}{\partial x},\quad Q=-\pi\int_0^R \frac{du}{dr}r^2dr.$$

Thus, we consider the equation

$$\frac{\lambda}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)=\frac{\rho C_p Q}{\pi R^2}\frac{\partial T}{\partial x}.$$

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